COMPLEX COEFFICIENT OF COMPLEXITY FOR GAME-TREE STRUCTURES IN DISCRETE OPTIMIZATION OF A GEAR PUMP HAVING TEETH UNDERCUTTING

Adam DEPTUŁA, Marian A. PARTYKA

Abstract: A gear pump optimization requires calculating volumetric, hydraulic and mechanical, as well as total efficiency. The pump efficiency optimization can be multi-criteria or mono-criterion. Assuming that the total pump efficiency is the goal function and the parameters- we are looking for- are values of construction and/or exploitation parameters, then optimization can be made separately for construction and exploitation parameters looking for the maximum efficiency value. There are already studies which analyze the rank of validity parameters *Q, n, p, M* using multi-valued and then complex logic decision. Original solution in this work is the application of a complex coefficient of complexity a game graphs.

Key words: gear pump after tooth undercutting, optimization, complex coefficient of complexity, dependence graphs, game- tree structures.

1.Introduction

Fluid-flow machines form a wide group of systems. The work of fluid-flow machines is most frequently based on two states: transient state (in which values of the system functions change in time) and steady state (the functions values do not change in time or change periodically). From among the displacement pumps, gear pumps are most commonly used (their share is estimated at about 60%) as energy generators in hydraulic drive systems. This is owing to their simple and compact design, operational reliability, high resistance to working medium pollution, high efficiency and small overall dimensions in comparison with other pumping units. Fluid flow energy generators are one of the principal components of any hydraulic system. In industry, external meshing gear pumps are most commonly used. Their share is estimated at about 50% [1, 2]. This is owing to their simple and compact design, operational reliability, high resistance to working medium pollution, high efficiency, small overall dimensions in comparison with other pumping units and low manufacturing cost. Moreover, gear pumps can work at considerable rotational speeds and in this respect they are better than the other types of displacement pumps. Owing to the above advantages, gear pumps have found widespread use in the drive, control and lubrication systems of machines and equipment. Various optimization algorithms, e.g. the systematic search method, the Monte Carlo method and the gradient method, are employed [2]. There are already studies which analyze the rank of validity parameters *Q, n, p, M* using multi-valued and then complex logic decision [3, 4].

Original solution in this work is the application of a complex coefficient of complexity for game graphs. Different graph solutions mean connections between input and output data as well as decision variables of the analyzed system (e.g. in the machine system). Game tree-structures from each graph vertex describe the decision making process and the space of the possible states of the analysed system [5, 6]. The obtained game structures differ in shape and properties. In order to choose a game structure with the lowest complexity, it is necessary to calculate a complex coefficient of complexity for all structures that are obtained.

Tree structure, with the lowest values of complexity level is the simplest structure. It should be noted that the complex coefficient of complexity of the structure is used in the description of multi-valued logical decision trees.

2. Gear pump after tooth root undercutting

The designed and built prototype pump has a three-plate structure shown schematically in Fig. 1. The front plate (1) is used for mounting the pump on the drive unit. The middle plate (2) contains gear wheels, slide bearing housings and suction and forcing holes for connecting to a hydraulic system. The whole construction is closed with a rear plate (3) [7, 8].

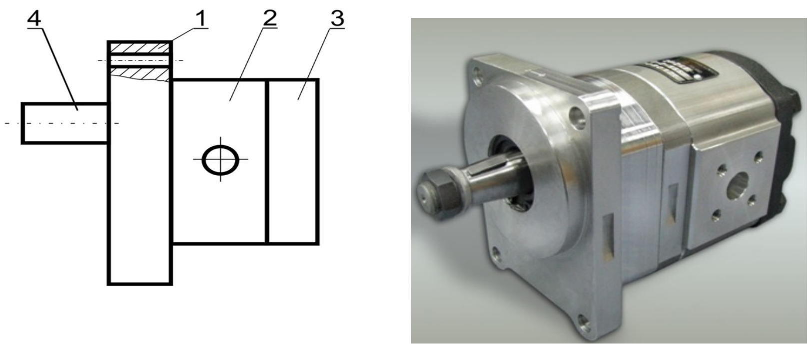

Fig. 1. Three-plate design of gear micropump with external meshing.
1 – front (mounting) plate, 2 – middle (rest) plate, 3– rear plate
4– driving shaft

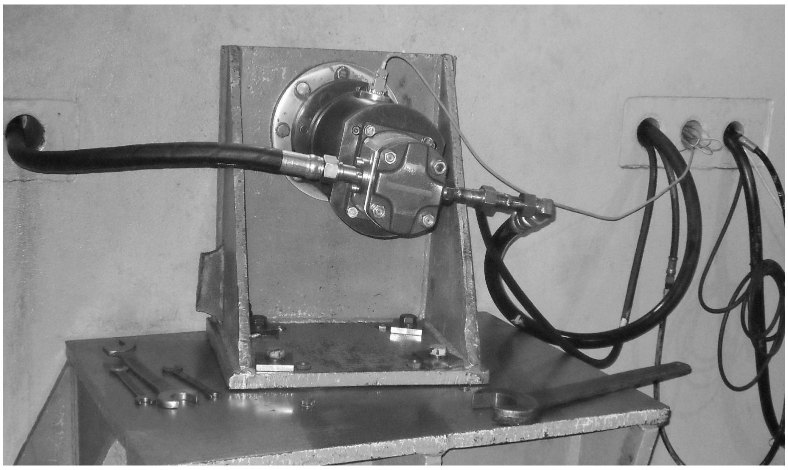

Fig. 2. The gear pump in the reverberation chamber

The tested prototype unit was designed in-house and manufactured by the Hydraulic Pumps Manufacturing Company Ltd. in Wrocław. The pump was designed having in mind the technological capacities of this company. Modification can be made by means of a cutting tool with the so-called prominence or by means of an appropriate choice of engagement correction. As far as the model of an involute tooth profile (Fig. 3) is concerned, it has been agreed that the representative top line will be moved towards the radius of the tool's foot with the tip clearance value equal to *lw* as a result of rounding or bevelling of the cutting edge. The tooth profile displacement of the correction value equal to $+x \cdot m0$ has also been taken into consideration [7, 8].

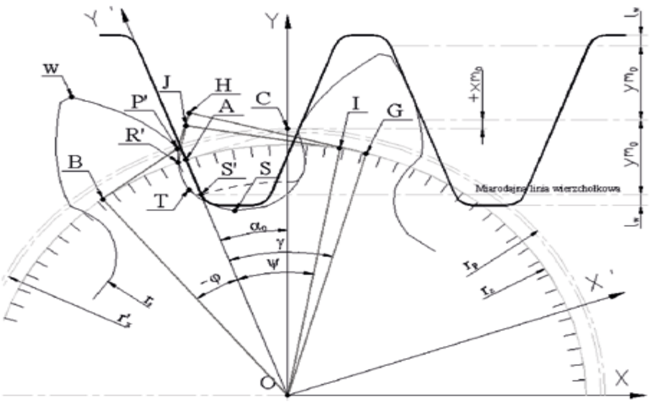

Fig. 3. The tooth undercutting by means of a trapezoidal profile rack [7, 8]

A gear pump optimization requires calculating volumetric, hydraulic and mechanical, as well as total efficiency. The pump efficiency optimization can be multi-criteria or mono-criterion. While looking for an optimal function value η_v, η_{hm}, η_c, the following arithmetic scopes of changes have been adopted: $\eta_v \geq 0{,}96$; $\eta_{hm} \geq 0{,}89$; $\eta_c \geq 0{,}86$. In [3], an analysis of a degree of importance of a gear pump construction parameters using multiple-valued logic decision trees was made for joined parameters $Q \wedge n$, p and M.

These values had been coded by means of logic decision variables for the needs of logic decision trees.

$n = 500$ [rpm] ~ 0; $n = 800$ [rpm] ~ 1; $n = 1000$ [rpm] ~ 2; $n = 1500$ [rpm] ~ 3; $n = 2000$ [rpm] ~ 4; $p_t = \approx 0$ [MPa] ~ 0; $p_t = 5$ [MPa] ~ 1 ; $p_t = 10$ [MPa] ~ 2; $p_t = 15$ [MPa] ~ 3; $p_t = 20$ [MPa] ~ 4; $p_t = 25$ [MPa] ~ 5; $p_t = 28$ [MPa] ~ 6; $p_t = 30$ [MPa] ~ 7; $Q_{rz} \in \langle 20,2;21,1 \rangle$ [l/min] ~ 0; $Q_{rz} \in \langle 34,2;34,9 \rangle$ [l/min] ~ 1; $Q_{rz} \in \langle 43,3;44,5 \rangle$ [l/min] ~ 2; $Q_{rz} \in \langle 65,5;67,3 \rangle$ [l/min] ~ 3; $Q_{rz} \in \langle 87,6;89,3 \rangle$ [l/min] ~ 4; $M \in \langle 2,0;47,0 \rangle$ [Nm] ~ 0; $M \in \langle 77,0;125,0 \rangle$ [Nm] ~ 1; $M \in \langle 138,0;182,0 \rangle$ [Nm] ~ 2; $M \in \langle 200,0;259,0 \rangle$ [Nm] ~ 3; where $M \in \langle 2,0;259,0 \rangle$ [Nm] and M_i ~ 0, 1, 2, 3.

The complex coefficient of complexity was calculated current for all multi-valued decision trees.

Table 1. Hydraulic measurement results. Arithmetic and logic values and the function of purpose [3, 8]

n [rpm]	p_t [Mpa]	Q_{rz} [l/min]	M [Nm]	Logic values n,	p_t,	Q_{rz},	M	η_v [%]	η_{hm} [%]	η_c [%]
500	0	21,1	2,0	0	0	0	0	94,6	0,0	0,0
	5	20,5	36,0		1		0	92,1	98,0	90,3
	10	20,3	77,0		2		1	91,3	91,8	83,8
	15	20,2	116,0		3		1	90,9	91,5	83,1
	20	20,2	156,0		4		2	90,9	90,7	82,4
	25	20,5	200,0		5		3	92,1	88,5	81,5
	28	20,6	218,0		6		3	92,5	90,9	84,1
	30	20,7	236,0		7		3	93,0	90,0	83,6
800	0	34,9	2,0	1	0	1	0	98,0	0,0	0,0
	5	34,7	38,0		1		0	97,5	92,8	90,5
	10	34,3	78,0		2		1	96,2	90,6	87,2
	15	34,2	118,0		3		1	96,0	89,9	86,3
	20	34,1	160,0		4		2	95,7	88,4	84,6
	25	34,5	202,0		5		3	97,0	87,6	85,0
	28	34,7	224,0		6		3	97,5	88,5	86,3
	30	34,8	240,0		7		3	97,8	88,5	86,5
1000	0	44,5	2,2	2	0	2	0	99,9	0,0	0,0
	5	44,1	38,0		1		0	99,1	92,8	92,0
	10	43,9	82,0		2		1	98,7	86,2	85,1
	15	43,4	124,0		3		1	97,4	85,6	83,4
	20	43,4	168,0		4		2	97,4	84,2	82,1
	25	43,4	208,0		5		3	97,4	85,1	82,9
	28	43,4	234,0		6		3	97,4	84,7	82,5
	30	43,3	249,0		7		3	97,2	85,3	82,9
1500	0	67,3	6,0	3	0	3	0	100,9	0,0	0,0
	5	66,8	42,0		1		0	100,0	84,0	84,0
	10	66,5	84,0		2		1	99,6	84,1	83,8
	15	66,1	125,0		3		1	99,1	84,9	84,1
	20	65,5	172,0		4		2	98,1	82,3	80,7
	25	65,7	210,0		5		3	98,4	84,2	82,9
	28	65,6	235,0		6		3	98,2	84,3	82,8
	30	65,5	255,0		7		3	98,1	83,3	81,7
2000	0	89,3	8,0	4	0	4	0	100,3	0,0	0,0
	5	89,0	47,0		1		0	100,0	75,0	75,0
	10	88,3	94,0		2		1	99,3	75,2	74,6
	15	88,0	138,0		3		2	98,8	76,9	76,0
	20	87,6	182,0		4		2	98,4	77,8	76,5
	25	88,0	214,0		5		3	98,8	82,7	81,7
	28	87,9	241,0		6		3	98,7	82,2	81,2
	30	87,8	259,0		7		3	98,6	82,0	80,9

Functions: η_v, η_{hm}, η_c are the criterion functions of purpose whereas parameters *n*, p_t, Q_{rz} are decision variables.

3. Complex coefficient of complexity for game-tree structures

Complex coefficient of complexity $L^K(G_i^{++})$ is applied to the description of the shape and properties of the game-tree structures previously obtained by the decomposition of the dependence graph on the *i*-th vertex. The level of structure's complexity of is determined by the complex coefficient of complexity $L^K(G_i^{++})$[9, 10, 11].

$$L^K(G_i^{++}) = \sum_{w \in W(L)} \frac{d(w_i)}{h(w_i)+1} + \frac{L}{\sum_{l \in L} \frac{1}{h_{l_i}}} \tag{1}$$

where:

$L^K(G_i^{++})$-complex coefficient of complexity of structure G_i^{++},
L- number of leaves for the *i-th* node branching ($\deg(w_i) \geq 2$),
h_{l_i} - amount (complexity) of the *i-th* leaf.
w_i -*i-th* node,
$d(w_i) = \deg(w_i)$ - degree of *i-th* node branching (amount of node branchings),
$h(w_i)$ - distance from the *i-th* node root,
$W(L)$ - set of all nodes.

The Figure 4 shows an example game-tree structure with different coefficients *L* and L^K.

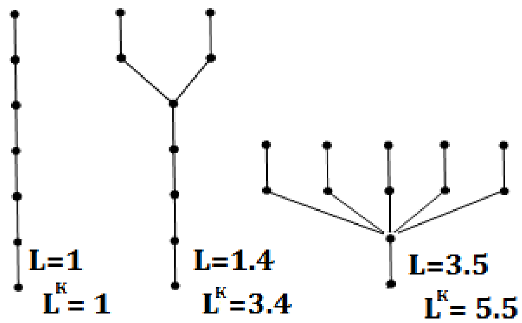

Fig. 4. Tree-game structures with different complexity coefficients *L* and L^K

It should be noted that the complex coefficient of complexity of the structure is used in the description of multi-valued logical decision trees [9].

Example 1

For a multiple-valued logic function $f(x_1, x_2, x_3)$ written by means of numbers KAPN: 000, 001, 002, 110, 003, 102, 004, 013, 014, 023, 124, 103 there is one MZAPN after the application of the Quine–McCluskey algorithm based on the minimization of individual partial multi-valued logic functions having 16 literals [12]:

$$f(x_1,x_2,x_3) = j_0(x_1)\big(j_0(x_2)+j_1(x_2)\big(j_3(x_3)+j_4(x_3)\big)+j_2(x_2)\big(j_3(x_3)+j_4(x_3)\big)\big)+ \\ j_1(x_1)\big(j_0(x_2)\big(j_2(x_3)+j_3(x_3)\big)+j_1(x_2)j_0(x_3)+j_2(x_3)j_4(x_3)\big)$$

The remaining ZAPN: $f(x_1,x_3,x_2)$, $f(x_3,x_2,x_1)$, $f(x_3,x_1,x_2)$, $f(x_2,x_3,x_1)$, $f(x_2,x_1,x_3)$ having 18, 22, 21, 20 and 17 literals. Figure 5 shows optimal multiple-valued logic trees.

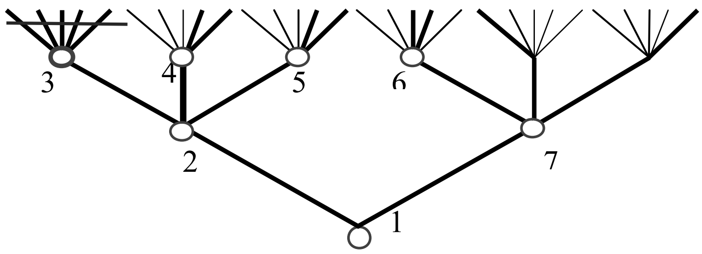

Fig. 5. Optimal multiple-valued logic tree for the logical function of Example 1

For each of the multi-valued logic trees after all possible cut-offs can be calculated complex coefficients of acomplexity of L^K. For optimal logic tree:

$$L^K(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3)=\left(\sum_{w\in W(L)}\frac{d(w_i)}{h(w_i)+1}+\frac{L}{\sum_{l\in L}\frac{1}{h_{l_i}}}\right)=\left(\frac{2}{0+1}+\frac{2}{\frac{1}{1}+\frac{1}{1}}\right)^1+\left(\frac{3}{1+1}+\frac{3}{\frac{1}{1}+\frac{1}{1}+\frac{1}{1}}\right)^2+\left(\frac{2}{2+1}+\frac{2}{\frac{1}{1}+\frac{1}{1}}\right)^4+$$

$$+\left(\frac{2}{2+1}+\frac{2}{\frac{1}{1}+\frac{1}{1}}\right)^5+\left(\frac{2}{2+1}+\frac{2}{\frac{1}{1}+\frac{1}{1}}\right)^6+\left(\frac{3}{1+1}+\frac{3}{\frac{1}{2}+\frac{1}{2}+\frac{1}{1}}\right)^7=10{,}59$$

For other multi-valued logic trees are: $L^K\left(f(x_1,x_3,x_2)\right)=11{,}82$ $L^K\left(f(x_3,x_2,x_1)\right)=15{,}3$ $L^K\left(f(x_3,x_1,x_2)\right)=17{,}2$ $L^K\left(f(x_2,x_3,x_1)\right)=16{,}26$ $L^K\left(f(x_2,x_1,x_3)\right)=11{,}64$

3. 1. Complex coefficient of complexity in discrete optimization of a gear pump after tooth undercutting

Tree structures described complex coefficient of complexity for each performance η_v, η_{hm}, η_c are shown in Figures 6-8. The optimal tree structures described complex coefficient of complexity for each performance are shown in Figures 9-11.

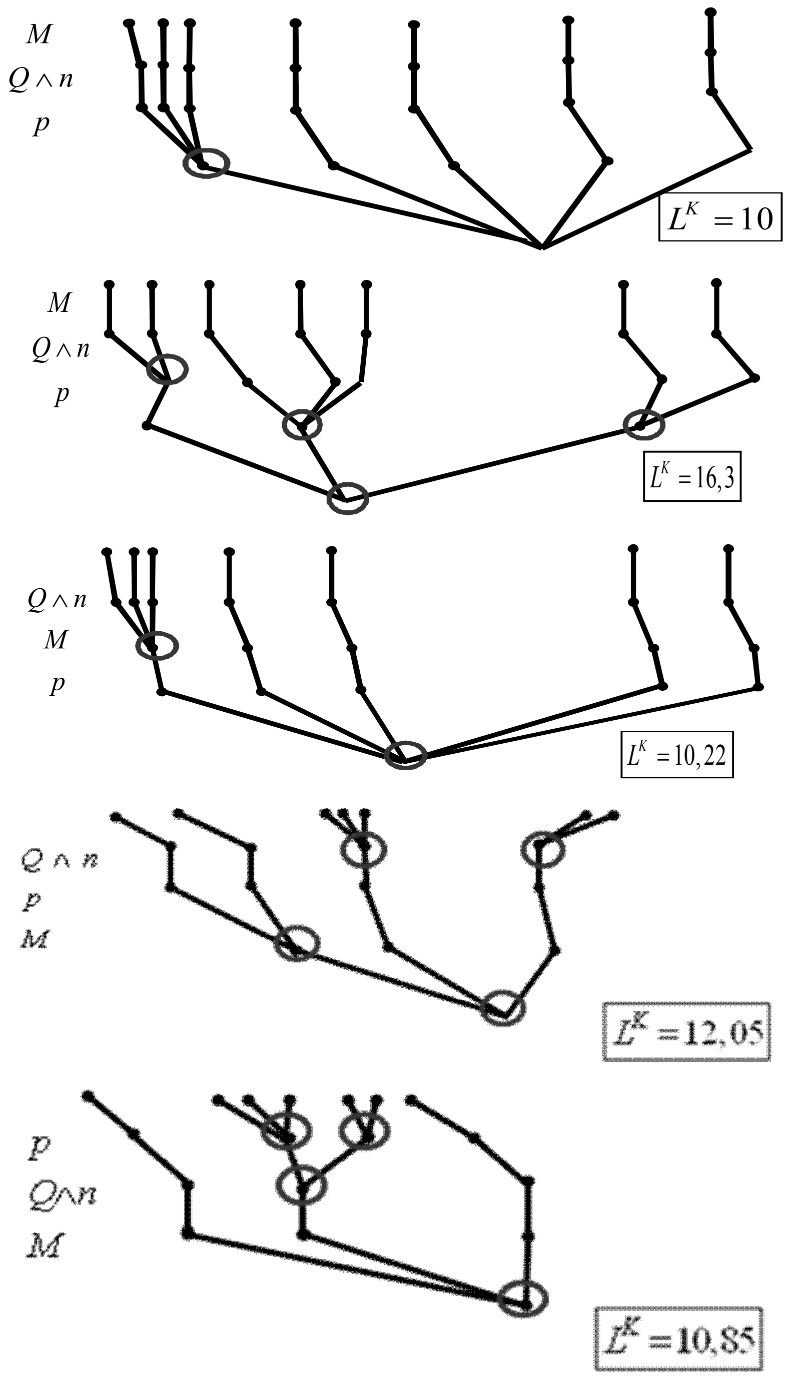

Fig. 6. Complex coefficients of complexity for efficiency η_c

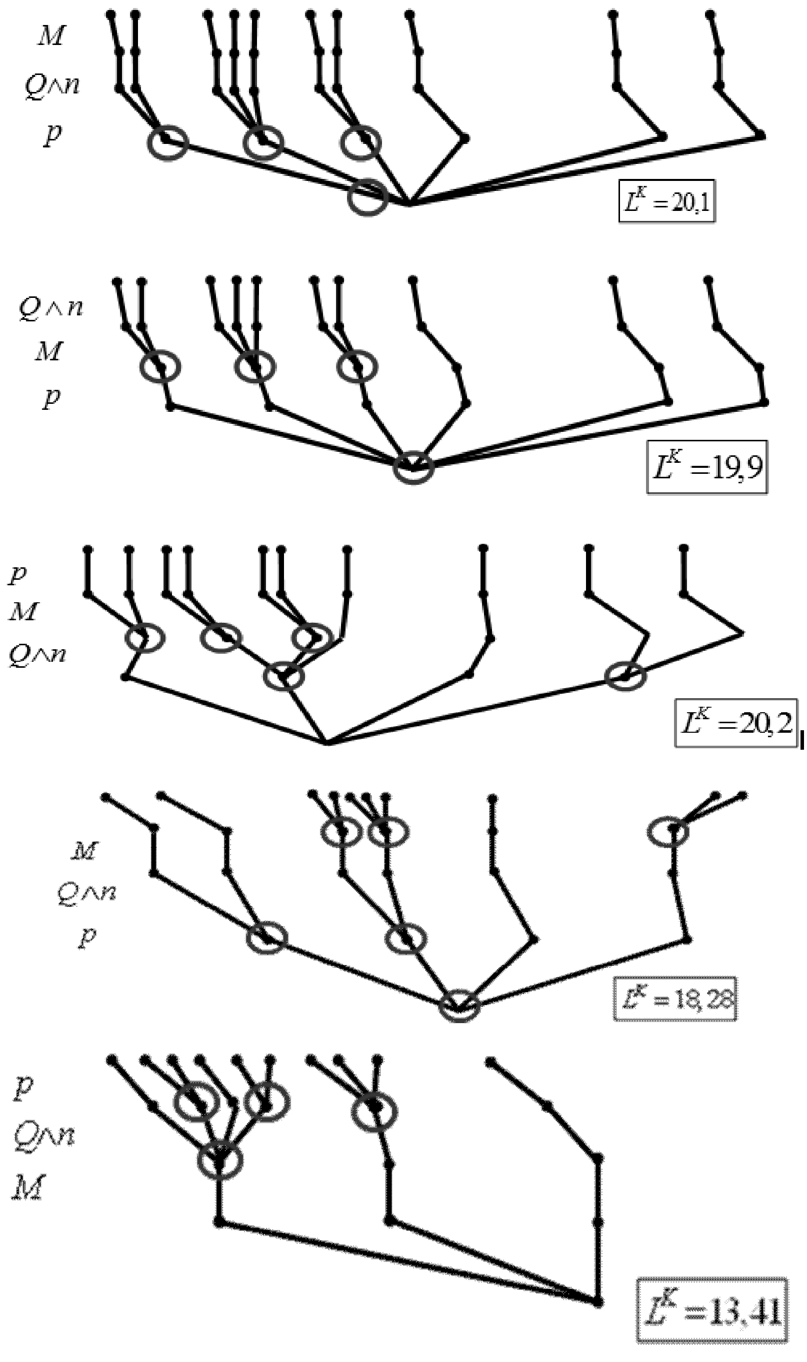

Fig. 7. Complex coefficients of complexity for efficiency η_{hm}

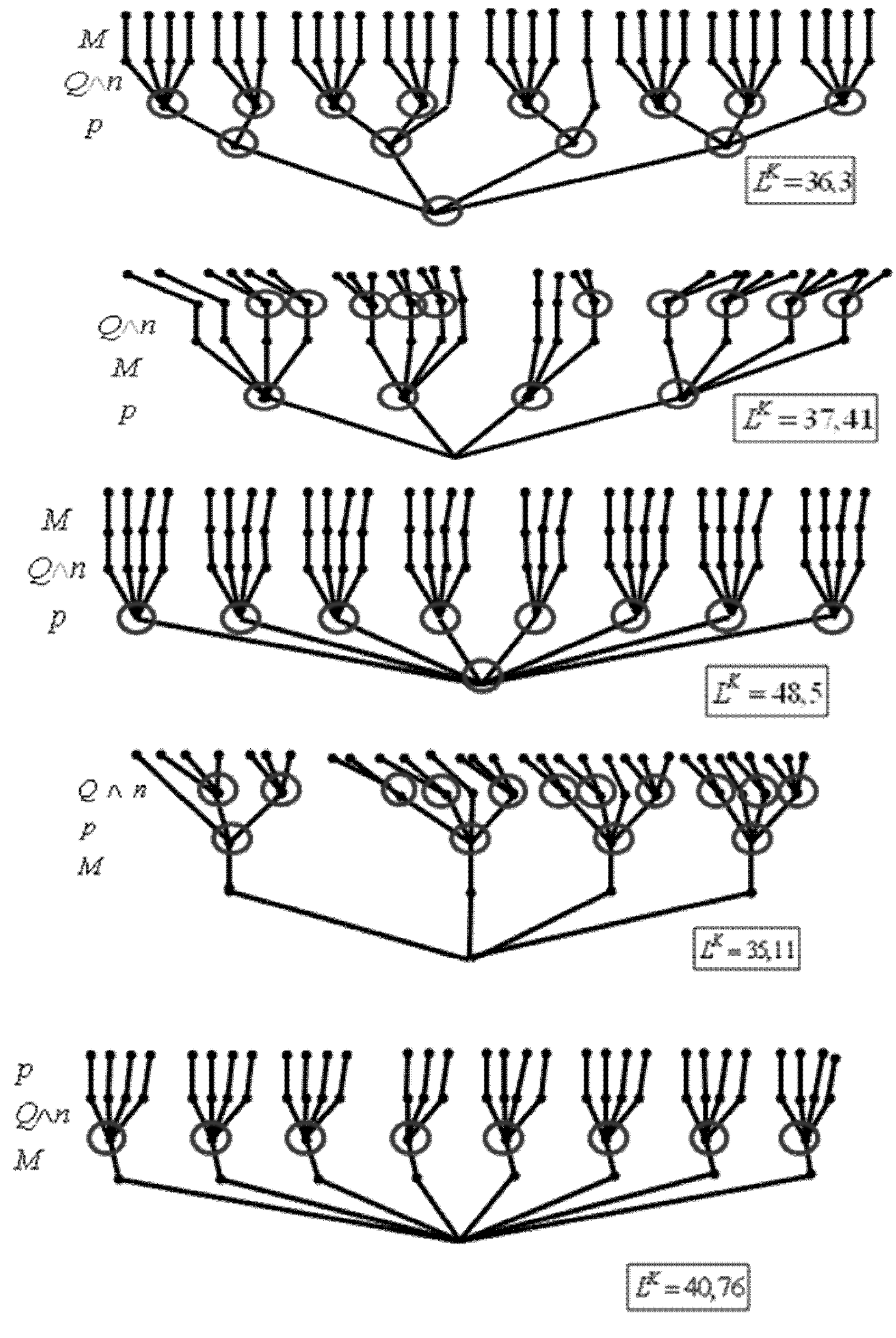

Fig. 8. Complex coefficients of complexity for efficiency η_v

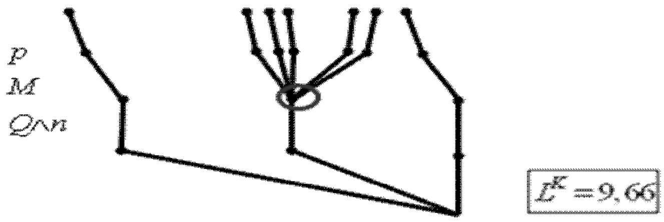

Fig. 9. Optimal complex coefficient of complexity for efficiency η_c

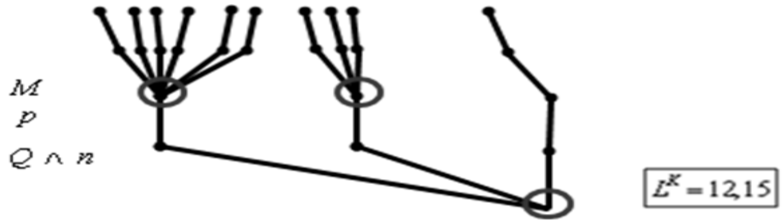

Fig.10. Optimal complex coefficient of complexity for efficiency η_{hm}

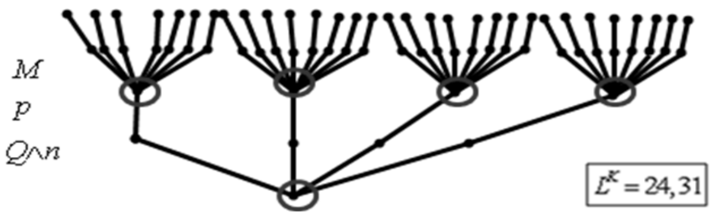

Fig.11. Optimal complex coefficient of complexity for efficiency η_v

For each performance η_v, η_{hm}, η_c, complex complexity of coefficients are as follows: For $\eta_c: L^K_{\eta_c}(p|Q\wedge n|M) = 10$, $L^K_{\eta_c}(p|M|Q\wedge n) = 10.22$, $L^K_{\eta_c}(Q\wedge n|p|M) = 9.66$, $L^K_{\eta_c}(M|p|Q\wedge n) = 16{,}3$, $L^K_{\eta_c}(Q\wedge n|M|p) = 10{,}85$, $L^K_{\eta_c}(M|Q\wedge n|p) = 12{,}05$. For $\eta_{hm}: L^K_{\eta_{hm}}(p|Q\wedge n|M) = 20.1$, $L^K_{\eta_{hm}}(p|M|Q\wedge n) = 19.9$, $L^K_{\eta_{hm}}(M|p|Q\wedge n) = 20.2$, $L^K_{\eta_{hm}}(M|Q\wedge n|p) = 18{,}28$, $L^K_{\eta_{hm}}(Q\wedge n|p|M) = 12.15$, $L^K_{\eta_{hm}}(Q\wedge n|M|p) = 10{,}85$. For $\eta_v: L^K_{\eta_v}(p|Q\wedge n|M) = 48{,}5$, $L^K_{\eta_v}(p|M|Q\wedge n) = 36.3$, $L^K_{\eta_v}(M|Q\wedge n|p) = 37{,}41$, $L^K_{\eta_v}(Q\wedge n|M|p) = 35.11$, $L^K_{\eta_v}(Q\wedge n|p|M) = 24.31$, $L^K_{\eta_v}(M|p|Q\wedge n) = 40.76$.

Among the optimal multiple-valued logic trees (the smallest number of the real branches) the optimal tree is highlighted, ie the lowest value of complex coefficient of the complexity [3]. In particular, the complex coefficient of complexity for each performance of the Figures 9-11designate identical optimal complex multiple-valued logic trees (according to work [4]) for the joint and the distributed parameters $M \wedge p$ as shown in Figures 12-14.

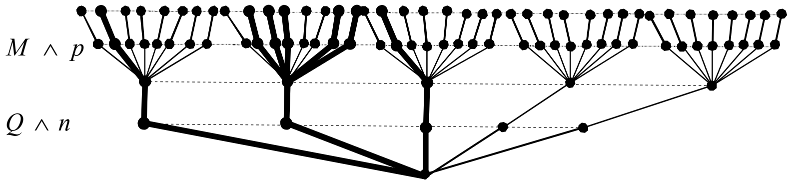

Fig.12. Optimal multiple-valued complex logic tree for the η_c efficiency [4]

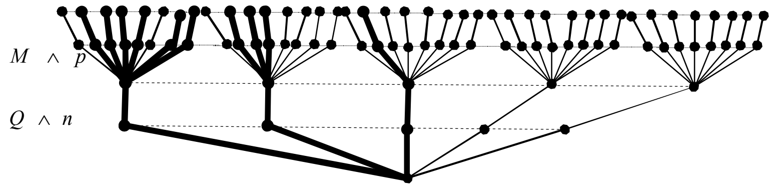

Fig.13. Optimal multiple-valued complex logic tree for the η_{hm} efficiency [4]

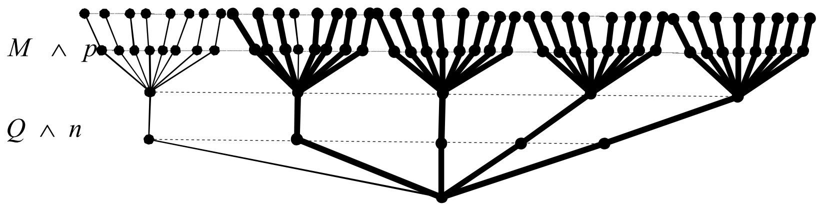

Fig.14. Optimal multiple-valued complex logic tree for the η_v efficiency [4]

For example, for the optimal multiple-valued complex logic tree in figure 12, the optimum η_c efficiency (90,3%; 90,5%; 87,2%; 86,3%; 86,3%; 86,5%; 92,0%) are obtained for the code parameter changes $Q \wedge n$ and $M \wedge p$: 0∧0 and 0∧1 - 90%,3; 1∧1 and 0∧1 - 90,5%; 1∧1 and 2∧1- 87,2%; 1∧1 and 3∧1-86,3%;1∧1 and 6∧3-86,3%;1∧1 and 7∧3-86,5%;2∧2 and 1∧0- 92%. The numerical values given coded entries presented in Table 1.

4. Conclusion

Game graphs make it possible to analyse the so-called "connected" decisions. Results obtained after the first decision have an influence on subsequent decisions. This is why they make it possible to make dynamic models. The graph distribution from any vertex leads to a tree game structure. Therefore, for each structure is necessary to calculate the coefficient of complexity. Tree structure, with the lowest values of complexity level is the simplest structure. It's very important that the complex complexity of coefficients of the structure is used in the description of multi-valued logical decision trees. For discrete optimization of a gear pump after tooth undercutting the complex complexity of coefficient for each performance designate identical optimal complex multiple-valued logic trees.

This important property of a complex complexity of coefficients for optimal multi-valued logic trees will allow for further research to develop a method for the direct determination of the optimal multi-valued logic trees of the dependency graph describing the system engineering.

References:

1. Francis J., Betts P. L.: *Modelling Incompressible Flow in a Pressure Relief Valve*.
2. Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering, Vol. 211, No. 2/1997, 83-93.
3. Huang J.K., Chang R.W., Lian Ch. W.: *An Optimization Approach to the Displacement Volumes for External Spur Gear Pumps,* Materials Science Forum Vol. 594 (2008).
4. Osiński P., Deptuła A., Partyka M.A.: *Discrete optimization of a gear pump after tooth root undercutting by means of multi-valued logic trees*, Archives of Civil and Mechanical Engineering, Volume 13, Issue 4, December 2013, Pages 422 431.
5. Deptuła A., Partyka M.A.: *Discrete optimization of a gear pump after tooth undercutting by means of complex multi-valued logic trees*. XVI Konferencja Innowacje w Zarządzaniu i Inżynierii Produkcji, Zakopane 2013, Pol. Towarz. Zarz. Prod. PTZP.
6. Deptuła A., Partyka M.A.: *Application of game graphs in optimization of dynamic system structures*, International Journal of Applied Mechanics and Engineering, 2010, vol.15, No.3, pp. 647-656.
7. Deptuła A., Partyka M.A.: Application of dependence graphs and game trees for decision decomposition for machine systems, Journal of Automation, Mobile Robotics & Intelligent Systems, Vol. 5, No. 4, 2011, pp. 17-26, ISSN 1897- 8649.
8. Kollek W., Osiński P.: Modelling and design of gear pumps.Wydawnictwo PWr, Wrocław 2009.
9. Osiński P.: Impact of the tooth root undercutting on hydraulic and acoustic properties of gear pump Raport PRE nr 14/2005 Politechnika Wrocławska, Wrocław 2005.
10. Deptuła A.: Coefficient of the structure complexity for multi-valued decision logic trees XLI Konf. Zast. Mat., Zakopane 2012, Inst. Mat. PAN, Warszawa 2012.
11. Deptuła A.: Determination of game-tree structures complexity level in discrete optimization of machine systems, International Masaryk Conference for Ph. D. students and young researches. December 12-16, 2011. Hradec Kralove, Czech Republic.
12. Deptuła A.: Complex coefficient of complexity for game-tree structures in discrete optimization of a gear pump after tooth undercutting. XLII Konf. Zast. Mat., Zakopane 2013, Inst. Mat. PAN, Warszawa 2013.
13. Partyka M. A., The Quine- Mc Cluskey minimization algorithm of individualmultiple-valued partial functions for digital control systems, 3rd Inter. Confer. Syst. Engin., Wright State University, Dayton 1984.

Prof. Marian A. PARTYKA,
MSc Eng. Adam DEPTUŁA
Opole University of Technology
Department of Knowledge Engineering
75 Ozimska Street, 45-370 Opole, Poland
phone/fax: (+48 77) 449 8845
e-mail: a.deptula@po.opole.pl